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Geometric Sequences

Unit 7 Lesson 8

Students will be able to:

Solve problems involving geometric sequences.

Key Vocabulary:

- Sequence
- Progression
- First Term
- nth Term
- Common Ratio
- Sum of Geometric Sequence

DEFINITION FOR GEOMETRIC SEQUENCE

A **geometric progression** is a sequence of numbers in which each term, after the first, is obtained by multiplying the preceding number by a constant called **common ratio**. The elements of a sequence is called **terms**.

NOTATIONS FOR GEOMETRIC SEQUENCE:

a = first term

r = common ratio

n = number of terms

a_n = nth term

S = sum of geometric sequence

Formula of Geometric Sequence:

Terms of geometric sequence are presented by:

$$a, ar, ar^2, ar^3, \ldots, ar^{n-1}$$

nth formula:

$$a_n = ar^{n-1}$$

Formula of Geometric Sequence:

The sum of the terms of geometric sequence S,

Multiply by r,

Subtract,

$$
\begin{array}{rl}
S = & a + ar + ar^2 + \ldots + ar^{n-1} \\
(-)\quad S(r) = & \quad\;\; ar + ar^2 + \ldots + ar^{n-1} + ar^n \\
\hline
S(1-r) = & a - ar^n
\end{array}
$$

First formula of the sum of the terms of geometric sequence S,

$$S = \frac{a - ar^n}{1-r} \text{ or } S = \frac{a(1-r^n)}{1-r}, r \neq 1$$

Formula of Geometric Sequence:

from

$$S = \frac{a - ar^{n}}{1 - r} = \frac{a - r(ar^{n-1})}{1 - r}$$

Second formula of the sum of the terms of geometric sequence S

$$S = \frac{a - ra_{n}}{1 - r}, r \neq 1$$

Formula of Geometric Sequence:

If **a** is the first term and **r** is the common ratio of an infinite geometric sequence, and if $|r| < 1$, then the sum of the terms of the sequence is given by the formula

$$S = \frac{a}{1-r}, r \neq 1$$

Sample Problem 1: The first three terms of a geometric sequence are given. Find the next three terms of the following geometric sequence.

$1. 2, 4, 8, \ldots.$

$2. -3, 1, -\frac{1}{3}, \ldots$

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$1. 2, 4, 8, \ldots.$

Common ratio : 2

$$8(2) = 16$$

$$16(2) = 32$$

$$32(2) = 64$$

$2. -3, 1, -\frac{1}{3}, \ldots$

Common ratio : -1/3

$$\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) = \frac{1}{9}$$

$$\left(\frac{1}{9}\right)\left(-\frac{1}{3}\right) = -\frac{1}{27}$$

$$\left(-\frac{1}{27}\right)\left(-\frac{1}{3}\right) = \frac{1}{81}$$

Sample Problem 2: Solve the following problem involving the nth term of geometric sequence.

4. find the 6th term in the Geometric progression 3, 6, 12,....

5. find the eight term in the geometric sequence 243, 81, 27,....

Sample Problem 2: Solve the following problem involving the nth term of geometric sequence.

4. find the 6th term in the Geometric progression 3, 6, 12,....

Given: $a = 3; r = 2; n = 6$

Solution:

$$a_6 = 3(2)^{6-1} = 3(2)^5 = 3(32) = 96$$

Sample Problem 2: Solve the following problem involving the nth term of geometric sequence.

5. find the eighth term in the geometric sequence 243, 81, 27,....

Given: $a = 243; r = \frac{1}{3}; n = 8$

Solution:

$$a_8 = 243\left(\frac{1}{3}\right)^{8-1} = 243\left(\frac{1}{3}\right)^{7} = 243\left(\frac{1}{2187}\right) = \frac{1}{9}$$

Sample Problem 3: Find the fifth term of the following geometric sequence given their first term and the common ratio.

5. $a = 3;\ r = 2$

6. $a = 5;\ r = -1$

Sample Problem 3: Find the fifth term of the following geometric sequence given their first term and the common ratio.

5. a = 3; r= 2

Solution:

$$a_5 = (3)(2)^{5-1}$$
$$= (3)(2)^4$$
$$= (3)(16)$$
$$= 48$$

6. a= 5; r = -1

Solution:

$$a_5 = (5)(-1)^{5-1}$$
$$= (5)(-1)^4$$
$$= (5)(1)$$
$$= 5$$

Sample Problem 4: Solve problem involving the sum of geometric sequence.

7. solve for the sum of the first 6 term of a geometric sequence -1, -5, 25,....

8. the third term of a geometric sequence is 20 and the fifth term is 80 what is the second term?

9. The population of an island increases by 10% each year. If the initial population is 500, what is the expected population after 5 years?

10. a certain type of bacteria doubles in number every day. if an initial amount of 1,000 bacteria was counted, what is the population after 5 days?

Sample Problem 4: Solve problem involving the sum of geometric sequence.

7. solve for the sum of the first 6 terms of a geometric sequence -1, -5, 25,....

Given: $a=-1; r=5; n=6$

Solution:

$$S=(-1)\left(\frac{1-(5)^6}{1-5}\right) = (-1)\left(\frac{1-15625}{-4}\right) = (-1)\frac{-15624}{-4}$$

$$=\frac{15624}{-4} \qquad =-3906$$

Sample Problem 4: Solve problem involving the sum of geometric sequence.

8. the third term of a geometric sequence is 20 and the fifth term is 80 what is the second term?

Equation: $20 = ar^{3-1} = ar^2; 80 = ar^{5-1} = ar^4$

Solution:

Find r: $\frac{80}{20} = \frac{ar^4}{ar^2};$

$4 = r^2; \sqrt{4} = \sqrt{r^2}$

$r = 2$

Find a: $20 = ar^2;$

$20 = a(2)^2; 20 = 4a;$

$\frac{20}{4} = \frac{4a}{4};$

$a = 5$

Find 2nd term:

$a_2 = ar$

$= (5)(2)$

$= 10$

Sample Problem 4: Solve problem involving the sum of geometric sequence.

9. The population of an island increases by 10% each year. If the initial population is 500, what is the expected population after 5 years?

Given: $a = 500, r = 0.1 + 1 = 1.1, n = 5$

Solution:

$$S = (500)\frac{\left(1-(1.1)^5\right)}{(1-1.1)} = (500)\frac{(1-1.61051)}{(-0.1)} = (500)\frac{(-0.61051)}{(-0.1)}$$

$$= \frac{-305.255}{-0.1} \quad = 3052.55 or 3052$$ People after 5 years

Sample Problem 4: Solve problem involving the sum of geometric sequence.

10. a certain type of bacteria doubles in number every day. if an initial amount of 1,000 bacteria was counted, what is the population after 5 days?

Given: $a = 1000; r = 2; n = 5$

Solution:

$$S = (1000)\frac{(1-(2)^5)}{(1-2)} = (1000)\frac{(1-32)}{-1} = (1000)\frac{-31}{-1}$$

$$= \frac{-31000}{-1} \quad = 31000$$ bacteria after 5 days

Sample Problem 5: Find the sum of a infinite geometric sequence

11. Find the sum of the terms of the infinite geometric sequence 125, 25, 5,…..

12. Find the sum of the infinite geometric sequence 64,-4,1/4,…

Sample Problem 5: Find the sum of a infinite geometric sequence

11. Find the sum of the terms of the infinite geometric sequence 125, 25, 5,.....

Given: $a = 125; r = \frac{1}{5}$

Solution:

$$S = \frac{a}{1-r} = \frac{125}{\left(1-\frac{1}{5}\right)} = \frac{125}{\frac{4}{5}} = \frac{5}{4}(125) = \boxed{\frac{625}{4}}$$

Sample Problem 5: Find the sum of a infinite geometric sequence

12. Find the sum of the infinite geometric sequence 64,-4,1/4,…

Given: $a = 64; r = -\frac{1}{16}$

Solution:

$$S = \frac{a}{1-r} = \frac{64}{\left(1-\left(-\frac{1}{16}\right)\right)} = \frac{64}{\frac{17}{16}} = \frac{16}{17}(64) = \boxed{\frac{1024}{17}}$$